7.10 Different Perspectives in Animal Breeding

In discussing profit equations, so far we have given little attention to the following four related issues that refer to definition of the breeding objective:

1) *From what perspective* should the benefits of genetic improvement be viewed. As noted in the introduction to this Chapter, we might view genetic improvement, and hence profit, from the view of the breeder/breeding company, the producer, the processor, the consumer, the whole industry, or some other view.

2) Should profit be expressed *per farm, per animal, or per unit of product*?

3) Should the breeding objective be to *maximize profit* (i.e. R-C) or to *maximize economic efficiency* (i.e. R/C), where R = total returns and C = total costs. Note that maximizing R/C is equivalent to minimizing C/R.

4) Should the breeding objective be defined *per farm, per animal, per unit of product, per unit of an input factor, or subject to any other constraint*?

It was Moav (1973) who first noted that different perspectives can yield different profit functions and different absolute and relative economic weights in the aggregate genotype. Subsequent authors have discussed this problem, and we illustrate it here with the example provided by Brascamp, Smith and Guy (1985).

Imagine a meat production enterprise consisting of $N$ breeding females, and producing $n$ offspring for slaughter each year. A simple profit function for the production enterprise could take the form,

$$P = N(nwr - nc_1d - c_2)$$

where $w$ is the weight of meat produced per offspring, $r$ is the returns per unit product, $d$ is the number of days to slaughter, $c_1$ the cost per day, and $c_2$ the cost of maintaining each female for one year. There are three traits under genetic control, $n$, $d$ and $w$.

Consider four different perspectives:

1) *profit per enterprise*: the viewpoint of the producer with potentially unlimited space for breeding females;
2) *profit per breeding female*: the viewpoint of the producer with a fixed number of breeding females;
3) *profit per slaughter progeny*: the viewpoint of the processor buying slaughter animals and interested in minimizing the cost per head;
4) *profit per unit of product*: the viewpoint of the consumer interested in minimum price per unit product.

The argument behind perspectives 3) and 4) is that an initial increase in profits due to genetic improvement ultimately results in reduced prices as competition forces prices down, so that something close to the original profit margin prior to genetic improvement is attained.
The appropriate profit equations are shown in Table 7.1 along with the resulting economic weights obtained as the partial derivatives of the profit equation: 

\[ v_{ij} = \frac{\partial P_j}{\partial y_i} \]

where \( j \) indicates the perspective taken (1 to 4) and \( i \) indicates trait \( i \).

**Table 7.1** Profit equations and economic weights for four profit perspectives*.

<table>
<thead>
<tr>
<th>Perspective</th>
<th>Profit equation</th>
<th>Economic Weight, ( v_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( P_1 = N(nwr - nc_1d - c_2) )</td>
<td>( v_n )</td>
</tr>
<tr>
<td>Per enterprise</td>
<td>( N(wr - c_1d) )</td>
<td>( -Nnc_1 )</td>
</tr>
<tr>
<td>Per female</td>
<td>( P_2 = nwr - nc_1d - c_2 )</td>
<td>( wr - c_1d )</td>
</tr>
<tr>
<td>Per individual</td>
<td>( P_3 = wr - c_1d - \frac{c_2}{2} )</td>
<td>( \frac{c_2}{2n} )</td>
</tr>
</tbody>
</table>
| Per unit product | \( P_4 = r - \frac{c_1d}{w} + \frac{c_2}{wn} \) | \( \frac{c_2}{2n} \) | \( -\frac{c_1}{w} \) | \( \frac{1}{w^2} \left( c_1d + \frac{c_2}{n} \right) \)

* \( n \) = number of progeny per breeding female per year; \( w \) = weight of product per slaughter animal; \( r \) = returns per unit product (price); \( c_1 \) = cost per slaughter animal per day; \( c_2 \) = cost per breeding female per year; \( d \) = days to market for slaughter animals.

It is clear from Table 7.1 that the relative economic weights for \( n \), \( d \) and \( w \) are the same for perspectives 1 and 2, the absolute values differing only by a scaling factor, \( N \). Thus, these two perspectives would result in equivalent genetic progress. Relative economic weights for \( n \), \( d \), and \( w \) do differ for other perspectives. This is disturbing, since it implies that different perspectives in the industry would have different indexes (and hence different directions of genetic change). But, the same animals must serve all levels of the industry.

The question then is whether it is possible to develop a consistent selection strategy (i.e. a consistent set of economic values) that meets the objective from every perspective. The answer to this question is yes, provided some important assumptions are made. Five related approaches have been suggested to obtain consistent economic values (after Goddard 1998):

1) **Zero or normal profit** (Brascamp et al. 1985): change the economic model by including normal return on investment as a cost, such that current profit equals zero.

2) **Rescaling** (Smith et al. 1986): subtract from the change in profit that results from genetic change the increase in profit that is due to a change in scale of the enterprise.

3) **Fixed base of comparison**: restrict total returns, total costs, or total profit to be constant.

4) **Define the objective as economic efficiency** \((R/C)\) (Dickerson (1978).

5) **Scale optimization** (Amer and Fox, 1992): increase the scale of the production system until an optimum is reached for the new genetic level.
7.10.1 Zero or Normal Profit

One branch of economic theory predicts that in stable but competitive markets, profit obtained at each level of an industry settles down to the “normal profit”. Normal profit is the profit necessary for persons operating a given level of the industry to make a reasonable return on their investment in time and money. In this case, normal profit can be viewed as a necessary operating cost, and would appear on the right hand side of the profit equation as a cost of production, so that profit now equals zero.

If the four profit equations in Table 7.1 are rewritten as zero profit equations, i.e. $P_i$ is set equal to zero, a new set of economic weights can be derived and these are shown in Table 7.2. To illustrate how these economic weights are derived, consider the profit equation expressed per slaughter animal,

$$ P_3 = wr - c_1d - \frac{c_2}{n} $$

which, from Table 7.1, gives an economic weight for $n$ of

$$ v_n = \frac{c_2}{n} $$

With zero profit:

$$ P_3 = 0 = wr - c_1d - \frac{c_2}{n} $$

so that

$$ c_2 = n(wr - c_1d) $$

and substituting for $c_2$ in the expression for $v_n$ we get:

$$ v_n = \frac{wr - c_1d}{n} $$

which is the value in Table 7.2.

<table>
<thead>
<tr>
<th>Perspective</th>
<th>Profit equation</th>
<th>Economic Weight, $v_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per enterprise</td>
<td>$P_1 = N(nwr - nc_1d - c_2) = 0$</td>
<td>$N\bar{w}r - Nc_1\bar{d}$</td>
</tr>
<tr>
<td>Per female</td>
<td>$P_2 = nwr - nc_1d - c_2 = 0$</td>
<td>$\bar{w}r - c_1\bar{d}$</td>
</tr>
<tr>
<td>Per individual</td>
<td>$P_3 = wr - c_1d - \frac{c_2}{n} = 0$</td>
<td>$\bar{w}r - c_1\bar{d}$</td>
</tr>
<tr>
<td>Per unit product</td>
<td>$P_4 = r - \frac{c_1d}{w} - \frac{c_2}{wn} = 0$</td>
<td>$\bar{wr} - c_1\bar{d}$</td>
</tr>
</tbody>
</table>

In the case of zero profit, relative economic weights given in Table 7.1 for $n$, $d$, and $w$ are the same for all perspectives, with absolute values for $P_1$, $P_3$ and $P_4$ differing from those for $P_2$ by factors of $N$, $\frac{1}{n}$, and $\frac{1}{nw}$. Thus all perspectives would result in the same relative index weights and the same direction of genetic change.
Brascamp et al. (1985) went on to give a general proof that this was true of zero (or normal) profit equations whenever the profit function could be written in the form: \( P = \frac{f(y,k_1)}{g(y,k_2)} \)

where \( f \) is any function of genetically controlled traits, \( y \), and economic traits, \( k_1 \), and \( g \) is any function of \( y \) and a vector of constants, \( k_2 \).

The concept of zero profit should not be interpreted as meaning that there is no incentive for genetic improvement. Under normal profit, if all producers were able to form a unified cartel, they could agree that no one should practice genetic improvement and all would retain their current profit without the expense of genetic improvement. But, in a competitive market, those producers who practice genetic improvement will increase their profits above those who do not or who do so less effectively. The incentive for change can then be viewed as either the economic advantage of practicing improvement when others do not, or the economic opportunity cost of not practicing improvement when others are (and are hence causing reduced prices as their normal profit returns to pre-improvement levels).

One problem with the zero profit approach is that incentives for appropriate change may not occur in rigidly structured industries with different sectors pursuing their own sectional interests. For example, consumers may desire lean beef, producers of slaughter calves may get paid premiums for conformation or carcass quality, but sellers of weaner calves seldom get premiums on the genetic quality of calves to yield lean beef. While this situation is maintained, breeders of weaner calves may pursue economic goals quite different from the interests of the consumer.

### 7.10.2 Rescaling

In this section a method of deriving economic weights proposed by Smith et al. (1986) is outlined, along with some related extensions of their proposal. The original intention of the authors was to show how, given certain assumptions, a variety of different methods and perspectives that had hitherto been seen as conflicting, were actually equivalent. (As always, it is up to you to decide whether or not you feel that the methods are valid and under what situations you might feel happy applying them.)

We have just seen that when normal profit applies, economic weights are the same from all perspectives. What is introduced here are arguments for treating all costs as variable costs and the need for enterprise rescaling due to genetic changes in production.

#### 7.10.2.1 Fixed Versus Variable Costs

The definition of fixed and variable costs is important when deriving economic weights. If fixed costs exist, genetic increases in output can cause extra returns at the same fixed cost. Do fixed costs exist? When an enterprise is started up the answer is certainly no. The investment is geared to the level of production anticipated. Equally, when re-investment in an enterprise to change its scale occurs, that investment is geared to the level of production anticipated. Thus over long time periods, so called fixed costs are related to production (output). Similarly when summed
over many production units (the national perspective) or when investment is continuous, fixed costs are variable in relation to the level of production and size.

Another line of reasoning for considering all costs as being variable argues that, if genetic increases in output can be accommodated without a change in fixed cost, the original enterprise must not have been at maximum efficiency. Any selection, which is made to fill existing inefficiencies in production, will be of short-term value. Such selection will therefore be made at the opportunity cost of selecting for those traits that reduce costs per unit output.

### 7.10.2.2 Rescaling Concept

The second argument is that any profit of genetic change, which could have been made by rescaling (changing the size) of the enterprise should not be attributed to that genetic change. For example, consider a genetic change, which increases the output of lean meat from a swine enterprise. The producer might have achieved the same increase in output by increasing the size of his enterprise, probably by changing the number of swine, by 10%. The true net value of the genetic change is therefore the difference in profit due to a 10% increase in output per pig versus a 10% increase in enterprise size. The difference is the economic improvement due to reduced costs per unit output.

### 7.10.2.3 Derivations of Economic Weights with Rescaling to Equal Output Value

If you are unfamiliar with the relationship between partial differentials and small changes, refer to Appendix B2 before reading this section.

Consider a profit equation of the general form \( P = R - C \)

where \( P \) = profit, \( R \) = returns and \( C \) = costs. \( R \) and \( C \) may be any function of any number of trait values. Assume that the enterprise has a scaling factor, \( N \), such that rescaling produces equal proportional changes in \( R \) and \( C \). Note that this rescaling factor means that there are no fixed costs. \( N \) could be interpreted as the number of animals but does not have to be.

Given the definition of the scaling factor \( N \):

\[
\frac{1}{R} \frac{\partial R}{\partial N} = \frac{1}{C} \frac{\partial C}{\partial N}
\]

Consider a trait, \( y \). Genetic change in \( y \) will lead to a change in profit of

\[
\Delta P_1 = \Delta R - \Delta C
\]

which, for a small change in \( y \), \( \Delta y \), gives

\[
\Delta P_1 = \left[ \frac{\partial R}{\partial y} - \frac{\partial C}{\partial y} \right] \Delta y
\]

The change in profit from genetic improvement is the result of both a change in output (returns), \( \frac{\partial R}{\partial y} \Delta y \), and a change in costs, \( -\frac{\partial C}{\partial y} \Delta y \). As argued earlier, the increase in output could have been achieved without genetic improvement, by rescaling the enterprise. Let the enterprise be
rescaled by a small change in \( N \), \( \Delta N \), to match the change in output (returns) caused by genetic change. Change in profit in this situation would be

\[
\Delta P_2 = \left[ \frac{\partial R}{\partial N} - \frac{\partial C}{\partial N} \right] \Delta N
\]

and the net value of genetic change is

\[
\Delta P_3 = \Delta P_1 - \Delta P_2
\]

Equating the change in output from rescaling the enterprise to the change in output from genetic improvement, note that

\[
\frac{\partial R}{\partial N} \Delta N = \frac{\partial R}{\partial y} \Delta y
\]

and from

\[
\frac{1}{R} \frac{\partial R}{\partial N} = \frac{1}{C} \frac{\partial C}{\partial N}
\]

\[
\Rightarrow \quad \frac{\partial C}{\partial N} = \frac{C}{R} \frac{\partial R}{\partial N}
\]

Hence,

\[
\Delta P_2 = \left[ \frac{\partial R}{\partial y} - \frac{C}{R} \frac{\partial R}{\partial y} \right] \Delta y
\]

Substituting the previous equations we get,

\[
\Delta P_3 = \left[ \frac{C}{R} \frac{\partial R}{\partial y} - \frac{\partial C}{\partial y} \right] \Delta y
\]

Dividing by \( \Delta y \) to get the economic value of unit change in \( y \):

\[
v_y = \frac{C}{R} \frac{\partial R}{\partial y} - \frac{\partial C}{\partial y}
\]

**Example**

As an example, consider again the profit function: \( P = N(nwr - nc_1d - c_2) \)

In this case

\( R = Nnwr \)

and

\( C = N(nc_1d + c_2) \)

Note that for this profit function, all costs are variable in relation to the scaling factor \( N \), the number of animals. Thus, at the level of number of animals, there are no fixed costs and rescaling by changing \( N \) produces equal proportional changes in returns.

The traditional economic value for \( n \) is:

\[
\frac{\partial P}{\partial n} = N(\bar{w}r - c_i\bar{d})
\]

Thus, a change in \( n \) by \( \Delta n \) results changes profit by:

\[
\Delta P_1 = N(\bar{w}r - c_i\bar{d})\Delta n
\]

and output value by:

\[
\Delta R_1 = N\bar{w}r\Delta n
\]

Output value can, however also be increased by changing \( N \); a change by \( \Delta N \) results in the following change in output value:

\[
\Delta R_2 = \Delta N\bar{n}\bar{w}r
\]

And the following change in profit:

\[
\Delta P_2 = (\bar{n}\bar{w}r - \bar{n}c_i\bar{d} - c_2)\Delta N
\]
Setting $\Delta N$ to match the change in output from $\Delta n$:  
$$\Delta R_1 = N \bar{w}r \Delta n = \Delta R_2 = \Delta N \bar{w}r$$

Thus:
$$\Delta N = \frac{N}{\bar{n}} \Delta n$$

Then, subtracting the profit that could be obtained from rescaling the enterprise to match the change in output, the net value of the genetic change by $\Delta n$ is:

$$\Delta P_3 = \Delta P_1 - \Delta P_2$$
$$= N(\bar{w}r - c_i \bar{d}) \Delta n - (\bar{n} w r - \bar{n} c_i \bar{d} - c_2) \Delta N$$

Substituting $\Delta N = \frac{N}{\bar{n}} \Delta n$ gives:
$$\Delta P_3 = N(\bar{w}r - c_i \bar{d}) \Delta n - N(\bar{w}r - c_i \bar{d} - \frac{c_2}{\bar{n}}) \Delta n$$
$$= N \frac{c_2}{\bar{n}} \Delta n$$

Thus the economic value is:
$$v_n = N \frac{c_2}{\bar{n}}$$

Rescaling against alternative methods of increased output is not the only form of rescaling that can be achieved. Other, equally plausible, possibilities are to rescale against increased input value or increased profits.

Economic values for $w$ and $d$ can be derived similarly, resulting in: $v_w = N \frac{\bar{n} c_i \bar{d} \bar{c}}{\bar{w}}$

Since a change in $d$ only affects costs, the economic value for $d$ is not affected by rescaling:
$$v_d = -N \bar{n} c_i$$

### 7.10.3 Fixed Base of Comparison

(Dekkers)

As an alternative to rescaling to match the increase in output value (or input value, or profit), the enterprise might be rescaled so that total returns (output value), costs (inputs value), or profits remained constant. Economic weights can be derived for these situations by following a similar approach as outlined above. Using the example, as shown previously, for trait $n$, a change in $n$ by $\Delta n$ results in a change in output value by:

$$\Delta R_1 = N \bar{w}r \Delta n$$

whereas a change in $N$ by $\Delta N$ results in a change in output value by:

$$\Delta R_2 = \Delta N \bar{n} \bar{w}r$$

Forcing a change in $N$ such that output value remains unchanged when $n$ is changed by $\Delta n$, $\Delta N$ can be solved by setting $\Delta R_2 = -\Delta R_1$:

$$\Delta N \bar{n} \bar{w}r = -N \bar{w}r \Delta n$$

Thus:
$$\Delta N = -\frac{N}{\bar{n}} \Delta n$$

Changing $N$ by $\Delta N = -\frac{N}{\bar{n}} \Delta n$ results in a change in profit equal to:

$$\Delta P_2 = (\bar{n} \bar{w}r - \bar{n} c_i \bar{d} - c_2) \Delta N$$
Thus the net economic value if $n$ is changed by $\Delta n$ is:

$$\Delta P_3 = \Delta P_1 + \Delta P_2$$

$$= N(\bar{w}r - c_1\bar{d})\Delta n - N(\bar{w}r - c_1\bar{d} - \frac{c_2}{n})\Delta n$$

$$= N\frac{c_2}{n}\Delta n$$

and the economic value is:

$$v_n = N\frac{c_2}{n}$$

Note that this is equivalent to the economic value derived previously with rescaling to match changes in output. Thus, rescaling to fixed output value is equivalent to rescaling to match changes in output value. The same holds for rescaling to fixed input value or profit.

### 7.10.4 Economic Efficiency

Starting in the early 70’s, Dickerson argued that the only reasonable way of evaluating genetic change is by examining the effect of genetic change on the economic efficiency ratio, $\phi = R/C$, rather than on profit.

Using the previous example, economic values based on economic efficiency can be derived as follows:

$$\phi = R/C = \frac{nwr}{nc_1d + c_2}$$

$$v_n = \frac{\partial \phi}{\partial n} = \frac{\bar{w}r}{\bar{nc}_1\bar{d} + c_2} \cdot \frac{n\bar{w}r c_1\bar{d}}{(\bar{nc}_1\bar{d} + c_2)^2} = \frac{c_2\bar{w}r}{(\bar{nc}_1\bar{d} + c_2)^2}$$

Similarly, economic values for the other two traits can be derived to be equal to:

$$v_d = -\frac{c_1\bar{n}r \bar{w}r}{(nc_1\bar{d} + c_2)^2}$$

$$v_w = \frac{\bar{nr}}{\bar{nc}_1\bar{d} + c_2}$$

### 7.8.5 Comparision of Zero Profit, Rescaling, and Economic Efficiency

(Modified by Dekkers)

Table 7.3 summarizes economic values for the example when derived using the zero profit approach, rescaling to output value, or based on economic efficiency.
Table 7.3. Economic values for the example profit function using three alternative approaches

<table>
<thead>
<tr>
<th>Perspective</th>
<th>Economic Weight, (v_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(v_n)</td>
</tr>
<tr>
<td>Zero profit</td>
<td>(N(\overline{w}r - c_1 \overline{d}))</td>
</tr>
<tr>
<td></td>
<td>(= N \frac{c_2}{\overline{n}})</td>
</tr>
<tr>
<td>Rescaling to output value</td>
<td>(N \frac{c_2}{\overline{n}})</td>
</tr>
<tr>
<td>Economic efficiency</td>
<td>(\frac{c_2 \overline{w}r}{(n c_1 \overline{d} + c_2)^2})</td>
</tr>
</tbody>
</table>

Although the economic values derived using the three approaches appear quite different, the relative economic values are actually equal. To see this, note that with zero profit,

\[ N(\overline{n} \overline{w}r - \overline{n}c_1 \overline{d} - c_2) = 0 \]

and thus

\[ \overline{w}r - c_1 \overline{d} = \frac{c_2}{\overline{n}} \]

and

\[ \overline{n}r = N \frac{\overline{n} c_1 \overline{d} + c_2}{\overline{w}} \]

This makes the economic values for the zero profit approach equivalent to those for rescaling to output value. Also, note that economic values for rescaling to output and those based on economic efficiency differ by a factor \(\frac{C}{\phi} = \frac{N^2(\overline{n} c_1 \overline{d} + c_2)^2}{N \overline{n} \overline{w} r}\)

Thus, economic values based on zero profit, rescaling, and economic efficiency are equivalent.

Equivalencies of economic values based on rescaling to output value with those based on economic efficiency can also be shown in more general terms as follows. Recalling from section 7.10.2.3 that economic values with rescaling to output value can be derived as:

\[ v_y = \frac{C}{R} \frac{\partial R}{\partial y} - \frac{\partial C}{\partial y} \]

this can be rewritten as:

\[ v_y = \frac{1}{R} \left[ C \frac{\partial R}{\partial y} - R \frac{\partial C}{\partial y} \right] \]

\[ = \frac{C^2}{R} \frac{\partial (R/C)}{\partial y}, \text{ (since } \frac{d(R/C)}{dy} = \frac{1}{C} \frac{dR}{dy} - R \frac{dC}{dy} = \frac{C}{\phi} \frac{\partial \phi}{\partial y} \)
Table 7.4 shows similar equivalencies for rescaling to match or to fixed output value, input value, profit, or zero profit. We can note that $C$ and $\phi$ are the same for all traits and all situations. Thus the relative economic weights of each trait are the same for all situations. However the absolute economic weights differ, depending on the scaling factor, $\frac{C}{\phi}$, $C$ or $\frac{C}{\phi - 1}$.

Since $\phi = \frac{R}{C}$, $\frac{\partial \phi}{\partial y}$ is the rate of change in the ratio of $R:C$ with genetic change in $y$. In other words, the economic weight is always the rate of change in economic efficiency, scaled by a constant that depends on whether enterprise scaling is at the level of outputs, inputs or profit.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Economic Weight ($\nu_y$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economic efficiency</td>
<td>$\frac{\partial \phi}{\partial y}$</td>
</tr>
<tr>
<td>Zero profit</td>
<td>$\frac{C}{\phi} \frac{\partial \phi}{\partial y}$</td>
</tr>
<tr>
<td></td>
<td>Scaled</td>
</tr>
<tr>
<td></td>
<td>Fixed</td>
</tr>
<tr>
<td>Output value</td>
<td>$\frac{C}{\phi} \frac{\partial \phi}{\partial y}$ $\frac{C}{\phi} \frac{\partial \phi}{\partial y}$</td>
</tr>
<tr>
<td>Input value</td>
<td>$\frac{C}{\phi} \frac{\partial \phi}{\partial y}$ $\frac{C}{\phi} \frac{\partial \phi}{\partial y}$</td>
</tr>
<tr>
<td>Profit</td>
<td>$\frac{C}{\phi - 1} \frac{\partial \phi}{\partial y}$ $\frac{C}{\phi - 1} \frac{\partial \phi}{\partial y}$</td>
</tr>
</tbody>
</table>

$^1 C = \text{cost}, \phi = \frac{R}{C}$, and $R = \text{returns}$.

It is also important to note that rescaling to increased or fixed profit holds for any definition of the profit equation $P=R-C$. Thus, these economic weights also apply to the zero profit approach of Brascamp et al. (1985). Since these authors showed that all perspectives in the market were equivalent in this situation, the results derived above should apply to all perspectives and all forms of enterprise scaling considered.

Thus, under the initial assumptions of no fixed costs and the need to disallow increased profit that could be achieved by enterprise scaling, all conflicting perspectives and derivations presented in the scientific literature are shown to be equivalent.
7.10.6 Absolute Versus Relative Economic Weights

In the absence of rescaling, economic weights would normally be taken as the partial derivative of profit with respect to the trait in question, i.e. 
\[ v_y = \frac{\partial P}{\partial y} = \frac{\partial R}{\partial y} - \frac{\partial C}{\partial y} \]

Rescaling to output achieved by other means gives the following economic value:
\[ v_y = \frac{C}{R} \frac{\partial R}{\partial y} - \frac{\partial C}{\partial y} \]

Since \( C \) is generally less than \( R \) (i.e. the enterprise is profitable), absolute economic weights with rescaling are less than those without. The effect on relative economic weights depends on the variation in \( \frac{\partial R}{\partial y} - \frac{\partial C}{\partial y} \) between traits. In many cases, relative economic weights may be little affected. Thus, rescaling is often most important in deriving cost-benefits of animal breeding.

7.10.7 Some Problems with Rescaling

One possible criticism of the rescaling approach of Smith et al. (1986) is that when scaling to outputs or inputs, all inputs and outputs are considered only in relation to their contribution to profit at the current population mean. For example, consider the principle of scaling to fixed output in relation to the production of meat and wool from sheep. Assume that there is no limitation on inputs, i.e. that far more sheep could be reared if it were profitable to do so. As used by Smith et al. (1986), scaling to fixed output means scaling to fixed output value. It is assumed that the total output value of sheep from meat and wool is fixed. But this may not be true. It could well be that total production is limited by saturation of the market for meat or wool, but not both. Assume that the market for sheep meat is saturated so that excess production of meat has no market and makes sheep rearing beyond that point unprofitable. In that case, sheep that produced more wool at the same carcass weight would be more profitable; the economic value of wool should include all extra profits from increased output and not be scaled.

In practice it is probably very difficult to decide whether or not all traits are saturating the market. Both production systems and markets accommodate themselves to the type of animal available. Thus the method of Smith et al. (1986) achieves maximization of economic efficiency of the existing production/marketing status quo but does not consider the possibility of creating or expanding markets for some traits. Similar criticisms could be given for scaling to inputs. The current balance between breeding for current production or marketing systems and considering new balances among the traits remains to be explored.

7.10.8 Dealing With Quotas

One situation where scaling of all traits clearly runs into difficulties is when markets operate under legislated quotas on one or more but not all outputs. This could include legislated quota on production designed to manage markets, or quota on manure or mineral emissions from
production systems to limit the environmental impact of animal production (e.g. Gibson and Wilton 1998, Olesen et al. 2000).
Provided that such quota and the pricing systems that go with it have long term stability, a producer should allow for increased output opportunities for traits not under quota. The critical assumption is that the quota and pricing system will be around for a sufficiently long time. There would be little point in breeding for a quota system if the very act of producing genetic change caused modifications to the system, which partially negated those genetic changes.

Using the same notation as earlier, since total yield remains constant, for a small change, $\Delta y$, in initial output per animal $y$, 

$$Ny = (N + \Delta N)(y + \Delta y)$$

so that, ignoring second order terms, 

$$\Delta N = -\frac{\Delta y}{y} N$$

Enterprise profit before genetic change, is 

$$T = NP = N(R - C)$$

After a small genetic change in trait $y$, $\Delta y$, and scaling the enterprise so that quota is not exceeded, the new profit, $T_1$, is 

$$T_1 = (N + \Delta N) \left[ P + \left[ \frac{\partial R}{\partial y} - \frac{\partial C}{\partial y} \right] \Delta y \right]$$

which, ignoring second order terms, gives 

$$T_1 = T + P \Delta N + \left[ \frac{\partial R}{\partial y} - \frac{\partial C}{\partial y} \right] N \Delta y$$

and 

$$v_y = \frac{\Delta T}{N \Delta y} = \frac{\partial R}{\partial y} - \frac{\partial C}{\partial y} - \frac{P}{y}$$

All other traits, unconstrained by quota, can recoup the full value of increased output so that their economic weights are simply 

$$v_y = \frac{\partial R}{\partial y} - \frac{\partial C}{\partial y}$$

Note that we now have a situation quite different from that due to enterprise scaling relating to total inputs or outputs. Economic weights are now given by expressions of different form for traits under quota than for those not under quota. Neither equation for the economic values under quota can be written in the form 

$$v_y = a - \frac{\partial \phi}{\partial y}$$

and are thus not simply scaled measures of changes in economic efficiency.

Assuming that the initial enterprise is profitable, then $\frac{P}{y} > 0$ and the effect of scaling to quota is to reduce the economic value of the trait under quota relative to those for unconstrained traits. For highly profitable enterprises, $\frac{P}{y}$, the profit per unit output of $y$, can be large, so that the economic value of the trait under quota can be severely reduced, in some cases changing signs to become negative. In general, but not always, it appears that rescaling to allow for quota has a larger effect on relative economic weights than rescaling to total enterprise inputs or outputs.
It is interesting to note that, if \( R \) is a linear function of the trait under quota, then the equation for the economic value of the trait under quota becomes

\[
v_y = \frac{\partial C}{\partial y} + \frac{C(y)}{y} \cdot \frac{P^*}{y},
\]

where \( P^* \) is profit to returns and costs not dependent on \( y \), and \( \frac{C(y)}{y} \) is the average cost of production of trait \( y \). The economic weight of trait \( y \) under quota is therefore independent of the returns generated by that trait (i.e. the price).

**7.10.8.1 A Working Example**

Consider a linear profit equation constructed for comparing dairy cattle genotypes,

\[
P = 0.175 \text{ Milk Yield} + 5.00 \text{ Fat Yield} - 7.50 \text{ Labour} - 0.1 \text{ Feed Intake} - 1.0 \text{ Miscellaneous Costs}
\]

where \( P \) is expressed in $. As expressed in this problem, each trait is considered separately and has only returns or costs associated with it. Thus, milk and fat generate returns of 0.175 and 5.0 $/kg respectively, and labor, feed, and miscellaneous incur costs of 7.5 $/hr, 0.1 $/kg and 1 $/unit.

Population mean production and input levels are given in Table 7.5, along with the breeding values of two alternative sires, assumed known without error.

<table>
<thead>
<tr>
<th>Table 7.5 Population means and transmitting abilities of two sires as candidates for selection.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trait</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Milk (kg)</td>
</tr>
<tr>
<td>Fat (kg)</td>
</tr>
<tr>
<td>Labour (hr)</td>
</tr>
<tr>
<td>Feed (kg)</td>
</tr>
<tr>
<td>Miscellaneous</td>
</tr>
</tbody>
</table>

Ignoring constraints, at the population mean:

\[
P = 0.175 \times 5000 + 5.0 \times 190 - 7.5 \times 22 - 0.1 \times 5800 - 1.0 \times 100 = 980
\]

\[
R = 0.175 \times 5000 + 5.0 \times 190 = 1825
\]

\[
C = 7.5 \times 22 + 0.1 \times 5800 + 1.0 \times 100 = 845
\]
Consider economic weights scaled to output values: 

\[ v_y = \frac{C \, \hat{\phi}}{\phi} \frac{\partial \phi}{\partial y} = \frac{C \, \partial R}{R} \frac{\partial C}{\partial y} \]

and 

\[ \frac{C}{R} = \frac{845}{1825} = 0.463 \]

Resulting economic values are in Table 7.6.

The relative net economic value of sire A would be:

\[ T_A = 0.081 \, (00.0) + 2.315 \, (10) - 7.5 \, (-1) - 0.1 \, (-300) - 1.0 \, (0.0) = 60.65 \, \$ \text{/daughter lactation} \]

\[ T_B = 0.081 \, (1000) + 2.315 \, (20) - 7.5 \, (-2) - 0.1 \, (700) - 1 \, (10) = 32.30 \, \$ \text{/daughter lactation}. \]

In this example, sire A is more valuable than sire B. For the definition of economic weights operating here, it must be assumed that the initial enterprise, geared to the population mean, operates at maximum economic efficiency. There is no quota on any one output, though overall economic output is scaled, and there are no constraints on inputs. The number of animals in the initial enterprise does not enter the equation, it is assumed to be at an optimum level.

If there is a quota on milk volume which is recognized as stable, the appropriate economic weight for milk, \( v_1 \), would be 

\[ v_1 = \frac{\partial R}{\partial y} \frac{\partial C}{\partial y} - \frac{P}{y} \]

while for all other traits it would be 

\[ v_y = \frac{\partial R}{\partial y} - \frac{\partial C}{\partial y} \]

Resulting economic values are in Table 7.6 and using these values, net economic values of the two sires are:

\[ T_A = 87.5 \, \$ \text{/daughter lactation} \]

\[ T_B = -16.0 \, \$ \text{/daughter lactation}. \]

If the quota on volume operates, sire A is much superior to sire B. Again it must be remembered that for this method to be valid, it is assumed that the original production system is optimized and that inputs are not constrained.

Economic values if the quota were to apply to fat instead of volume are also given in Table 7.6., giving:

\[ T_A = 35.92 \, \$ \text{/daughter lactation} \]

\[ T_B = 76.84 \, \$ \text{/daughter lactation}. \]

In this case, sire B is considerably more valuable than sire A. Switching quotas from value to fat has a large effect on relative economic weights and consequent changes in sire selection.
Taking the profit equation at face value, and ignoring any constraints or need to rescale, yields economic weights \( v_y = \frac{\partial R}{\partial y} - \frac{\partial C}{\partial y} \) (see Table 7.6) giving \( T_A = 87.5 \)
\( T_B = 180. \)

**Table 7.6** Estimated economic values of traits and net economic values of sires A and B for different types of scaling, expressed as $ per daughter lactation.

<table>
<thead>
<tr>
<th></th>
<th>Not constrained</th>
<th>Scaled Output</th>
<th>Scaled to Quota on Volume</th>
<th>Scaled to Quota on Fat</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_1 )</td>
<td>0.175</td>
<td>0.463(0.175)-0.0 = 0.0810</td>
<td>0.175 - 980/5000 = -0.021</td>
<td>0.175</td>
</tr>
<tr>
<td>( v_2 )</td>
<td>5.0</td>
<td>0.463 (5.0) -0.0 = 2.315</td>
<td>5.0</td>
<td>5.0-980/190 = -0.158</td>
</tr>
<tr>
<td>( v_3 )</td>
<td>-7.5</td>
<td>0.0 (-7.5=-7.5)</td>
<td>-7.5</td>
<td>-7.5</td>
</tr>
<tr>
<td>( v_4 )</td>
<td>-0.1</td>
<td>0.0 (-0. =-0.1)</td>
<td>-0.1</td>
<td>-0.1</td>
</tr>
<tr>
<td>( v_5 )</td>
<td>-1.0</td>
<td>0.0 (-1.0 =-1.0)</td>
<td>-1.0</td>
<td>-1.0</td>
</tr>
<tr>
<td><strong>( T_A )</strong></td>
<td>87.50</td>
<td>60.65</td>
<td>87.5</td>
<td>35.92</td>
</tr>
<tr>
<td><strong>( T_B )</strong></td>
<td>180.0</td>
<td>32.3</td>
<td>-16.0</td>
<td>76.84</td>
</tr>
</tbody>
</table>

In this example, rescaling to allow for quotas has a very large effect on relative economic weights and absolute sire values. The effect is large because initial profit per cow, \( P \), was large so that the scaling factor, profit per unit yield of the trait under quota \( \frac{P}{y} \), is also large. This means that the potential to make improvements in economic efficiency of production of the trait under quota by genetically increasing output per cow is more than offset by losses in profit in other traits when reducing the number of cows to stay within quota. Usually, profit margins would not be so high and rescaling to quota would have a less dramatic effect.

**7.10.8.2 An Example of Re-Optimization with Constraints**

Consider a simple example of a dairy farm with a fixed quota, \( Q \), for production of a single output trait with production level per cow of \( y \), and zero payment for over quota production. The profit equation, recognizing the existence of the quota but ignoring the opportunity to optimize the system after genetic change, would be

\[
P = (R(y) - C(y)|Y \leq Q) - (C(y)|Y > Q)
\]
where $Y = Ny$ is total enterprise production of $y$ and $N$ is the number of cows. The first term in this equation is a combination of returns and costs functions that apply to under quota production, while the second term is the cost of producing $y$ over quota (since returns are zero over quota). Initially the enterprise would be optimized so that total production exactly fills the quota, i.e. $Y = Ny = Q$. If re-optimization is ignored, the economic weight for $y$ is found by differentiating that part of the profit equation that applies to over quota production (since all increases in output will be in excess of quota), i.e. $v_{\text{noopt}} = -\frac{\partial C}{\partial y}$, where $\frac{\partial C}{\partial y}$ is the cost of production per unit extra output.

The profit function could however be re-written to allow for optimization of the enterprise after genetic change. In this case the number of cows would be altered to stay within quota. The total enterprise profit, $T$, would be

$$T = N(R(y) - C(y))$$

and, since total production $Ny = Q$, $N = \frac{Q}{y}$, and

$$T = \frac{Q}{y} (R(y) - C(y)) = \frac{N_{\text{o}}y_o}{y} (R(y) - C(y))$$

giving an economic weight for $y$ for this optimized profit function of

$$v_{\text{opt}} = \frac{\partial f}{\partial y} \frac{1}{N_{\text{o}} \partial y} = \frac{Q}{N_{\text{o}} y^2} (R(y_o) - C(y_o)) + \frac{\partial R}{\partial y} - \frac{\partial C}{\partial y}$$

In this dairy cattle case with quota, $v_{\text{noopt}}$ would be negative and $v_{\text{opt}}$ positive and are clearly very different from each other.

The solution for $v_{\text{opt}}$ is identical to that given for the economic weight after allowing for scaling to stay within quota. The economic value, $v$, without rescaling is given as: $v = \frac{\partial R}{\partial y} - \frac{\partial C}{\partial y}$, and is clearly different from $v_{\text{noopt}}$ given here. The reason for this discrepancy is that when rescaling to quota was introduced, the initial profit equation ignored existence of the constraint. In the present example, existence of the constraint (quota) is recognized in the original profit function, but the change in management variables to optimize profit is ignored when deriving $v_{\text{noopt}}$.

Obtaining economic weights with rescaling to constant output (or quota on a single trait) involved allowing for the change in the number of animals to stay within the constraint. It should be clear, as done here, that this change could be incorporated directly into the profit function, so that profit is now defined as profit allowing for re-optimization of management to stay within a production constraint; and differentiation of the new profit equation leads directly to $v_{\text{opt}}$.

The importance of re-optimization of the management system should be examined on a case by case basis and will depend on the original formulation of the profit function (or model). While a
profit function ignoring re-optimization is often simpler, in general it seems safest to make sure that the profit function always allows for optimization of management to match genetic change.

As described above, quota restrictions can be incorporated into derivation of economic values through the concept of rescaling. With rescaling, economic values of the product under quota are equal to the economic value of the trait with unlimited output apart from subtracting a rescaling term. The rescaling term is equal to the average profit over fixed costs per unit of the product under quota. The same result is obtained when profit is described at the level of the whole enterprise (e.g., herd or country) instead of at the level of the individual animal. This equivalence holds provided dependence of number of cows in the enterprise on output per cow of the trait under quota is included in formulation of the profit function.

When quota is a tradable commodity, which is the case for most quota systems, the two approaches just discussed for dealing with quota may not appear sensible at the farm level because both assume an absolute restriction on output. Another approach to account for quota in the derivation of economic values is to charge interest on the market value of quota as a marginal cost for the product under quota. This more closely reflect market circumstances to which individual producers are exposed. This approach leads to economic values that are identical to economic values that are derived with rescaling when interest cost per unit of quota is equal to the average profit over fixed cost per unit of the product under quota, which is the term that is subtracted in derivation of economic values under rescaling. This condition is expected to hold when quota is traded on a free market that is in equilibrium.